MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

DEM5038 – ENGINEERING MATHEMATICS 3

(Group: E12)

7 MARCH 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **10** pages (**3** pages with **5** questions and **7** pages for appendix).

2. Answer **ALL** questions. All necessary working steps must be shown.

3. Write all your answers in the answer booklet provided.

Please answer ALL questions and show the necessary working. Total mark is equal to 100.

Question 1

Find the general solution to the following non-homogenous differential equation.

$$y'''-2y''-y'+2y = 6e^{2x} + 10e^{3x}$$ [15 marks]

[Total = 15 marks]

Question 2

a) Given one period of a periodic function as $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Find the Fourier series for the above function. [12 marks]

b) Find the Half-range Sine series for

$$f(x) = 2 - x, \qquad 0 \le x < 2$$ [8 marks]

[Total = 20 marks]

Question 3

a) Determine the Laplace transform of the following functions.

i. $2e^{-t}\cos 4t$ [2 marks]

ii. $3t^2e^{6t} - 5t^5$ [2 marks]

iii. $\int_0^t 4\sinh 3u\,du$ [2 marks]

iv. $f(t) = \begin{cases} 3, & 0 < t < 3 \\ t-3, & t > 4 \end{cases}$ [4 marks]

Continued...

b) Find the inverse Laplace transform for the following functions.

i. $\dfrac{12}{(s+3)^4}$ [2 marks]

ii. $\dfrac{1}{(s-4)^2+36}$ [2 marks]

iii. $\dfrac{4s^2+8s}{s^4}$ [2 marks]

iv. $\dfrac{5s}{s^2-25}$ [2 marks]

c) Find the solution to the following differential equation by using Laplace transform method.

$$y''+8y'+16y = 8e^{-2t} \qquad y(0) = 2,\ y'(0) = 0$$ [12 marks]

[Total = 30 marks]

Question 4

a) Table 1 shows the number of students sitting for each of the subject and there are no student taking more than one subject.

Table 1

	Mathematics	Economics	History
Boy	405	305	120
Girl	100	320	680

One student is selected randomly from this group of 1930 students. What is the probability that this student:

i. takes only History? [1 mark]

ii. is a boy and takes Mathematics? [2 marks]

iii. takes History, given that the student is a girl? [2 marks]

iv. takes Economics or History? [2 marks]

Continued...

b) An auto company claims that the mean value of cars owned by all Malaysian is RM 56,250. A sample of 500 cars gave a mean value of RM 54,200 with a standard deviation of RM12,375. Test at 1% significance level if the mean value of cars owned by all Malaysian is less than RM 56,250? [8 marks]

[Total = 15 marks]

Question 5

a) A study conducted shows that 40% of the batteries produced by a factory are not satisfactory. Using the binomial distribution function, find the probability that in a random sample of 10 batteries,

i. less than 2 batteries are not satisfactory. [2 marks]

ii. at least one battery is not satisfactory. [2 marks]

b) The life of a certain type of automobile tire is normally distributed with mean 34,000 km and standard deviation of 4000 km. What is the probability that such a tire lasts,

i. between 30,000 and 35,000 km? [2 marks]

ii. more than 30,000 km? [2 marks]

c) A clinic analyzed the age and glucose level in blood for 6 patients and the following results were tabulated in Table 2 as shown below.

Table 2

Patient	Age	Glucose Level
1	43	99
2	21	65
3	25	79
4	42	75
5	57	87
6	59	81

i. Determine the correlation coefficient between the age and glucose level in blood. [9 marks]

ii. Find the estimate regression line of y on x, $\hat{y} = a + bx$. [3 marks]

[Total = 20 marks]

End of Page.

Appendix

Quadratic formula $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Annihilator for special functions

Annihilator	
D^n	$1, x, x^2, x^3, x^4, \ldots\ldots, x^{n-1}$
$(D-a)^n$	$e^{ax}, xe^{ax}, x^2e^{ax}, x^3e^{ax}, x^4e^{ax}, \ldots\ldots, x^{n-1}e^{ax}$
$\left(D^2 - 2aD + (a^2+b^2)\right)^n$	$e^{ax}\cos bx, xe^{ax}\cos bx, x^2e^{ax}\cos bx, x^3e^{ax}\cos bx, \ldots\ldots, x^{n-1}e^{ax}\cos bx$ and $e^{ax}\sin bx, xe^{ax}\sin bx, x^2e^{ax}\sin bx, x^3e^{ax}\sin bx, \ldots\ldots, x^{n-1}e^{ax}\sin bx$

Fourier Series

$$F(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}xdx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}xdx$$

Integration Formula

$$\int \sin axdx = -\frac{1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int x\sin axdx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos axdx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin axdx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos axdx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

Linear Regression and Correlation

$$\hat{y} = a + bx$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \qquad SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$b = \frac{SS_{xy}}{SS_{xx}} \qquad a = \bar{y} - b\bar{x} = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Transformation of some functions

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$, s>0
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$ $n = 1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$, s>a
$t\,e^{at}$	$\frac{1}{(s-a)^2}$
y'	$sY(s)-y(0)$
y''	$s^2Y(s)-sy(0)-y'(0)$

$f(t)$	$L\{f(t)\}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$(1)u(t-a)$	$\frac{e^{-sa}}{s}$
$(t-a)u(t-a)$	$\frac{e^{-sa}}{s^2}$

Second Shift Theorem

$$L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$$
$$L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}$$

Binomial Probabilities $\binom{n}{x}p^x q^{n-x}$

Binomial Formulae when using Cambridge Statistical Table

Key Formulas (if $p \le 0.5$)

1. $P(X=r) = B(r) - B(r-1)$
2. $P(X \ge r) = 1 - B(r-1)$
3. $P(X > r) = 1 - B(r)$
4. $P(a \le X \le b) = B(b) - B(a-1)$
5. $P(X \le r) = B(r)$

Key Formulas (if $p > 0.5$)

1. $P(X=r) = P(Y=n-r) = B(n-r) - B(n-r-1)$
2. $P(X \ge r) = P(Y \le n-r) = B(n-r)$
3. $P(X \le r) = P(Y \ge n-r) = 1 - B(n-r-1)$
4. $P(a \le X \le b) = P(n-b \le Y \le n-a)$
   $= B(n-a) - B(n-b-1)$

Normal Distribution $Z = \frac{X-\mu}{\sigma}$

Hypothesis Testing $S_x = \frac{s}{\sqrt{n}}$ $Z = \frac{\bar{x}-\mu}{S_x}$

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 8$	$r = 7$	$n = 9$	$r = 0$	1	2	3	4	5	6	7	8
$p = 0{\cdot}01$		$p = 0{\cdot}01$	0·9135	0·9966	0·9999						
·02		·02	·8337	·9869	·9994						
·03		·03	·7602	·9718	·9980	0·9999					
·04		·04	·6925	·9522	·9955	·9997					
0·05		0·05	0·6302	0·9288	0·9916	0·9994					
·06		·06	·5730	·9022	·9862	·9987	0·9999				
·07		·07	·5204	·8729	·9791	·9977	·9998				
·08		·08	·4722	·8417	·9702	·9963	·9997				
·09		·09	·4279	·8088	·9595	·9943	·9995				
0·10		0·10	0·3874	0·7748	0·9470	0·9917	0·9991	0·9999			
·11		·11	·3504	·7401	·9328	·9883	·9986	·9999			
·12		·12	·3165	·7049	·9167	·9842	·9979	·9998			
·13		·13	·2855	·6696	·8991	·9791	·9970	·9997			
·14		·14	·2573	·6343	·8798	·9731	·9959	·9996			
0·15		0·15	0·2316	0·5995	0·8591	0·9661	0·9944	0·9994			
·16		·16	·2082	·5652	·8371	·9580	·9925	·9991	0·9999		
·17		·17	·1869	·5315	·8139	·9488	·9902	·9987	·9999		
·18		·18	·1676	·4988	·7895	·9385	·9875	·9983	·9998		
·19		·19	·1501	·4670	·7643	·9270	·9842	·9977	·9998		
0·20		0·20	0·1342	0·4362	0·7382	0·9144	0·9804	0·9969	0·9997		
·21		·21	·1199	·4066	·7115	·9006	·9760	·9960	·9996		
·22		·22	·1069	·3782	·6842	·8856	·9709	·9949	·9994		
·23		·23	·0952	·3509	·6566	·8696	·9650	·9935	·9992	0·9999	
·24		·24	·0846	·3250	·6287	·8525	·9584	·9919	·9990	·9999	
0·25		0·25	0·0751	0·3003	0·6007	0·8343	0·9511	0·9900	0·9987	0·9999	
·26		·26	·0665	·2770	·5727	·8151	·9429	·9878	·9983	·9999	
·27		·27	·0589	·2548	·5448	·7950	·9338	·9851	·9978	·9998	
·28		·28	·0520	·2340	·5171	·7740	·9238	·9821	·9972	·9997	
·29	0·9999	·29	·0458	·2144	·4898	·7522	·9130	·9787	·9965	·9997	
0·30	0·9999	0·30	0·0404	0·1960	0·4628	0·7297	0·9012	0·9747	0·9957	0·9996	
·31	·9999	·31	·0355	·1788	·4364	·7065	·8885	·9702	·9947	·9994	
·32	·9999	·32	·0311	·1628	·4106	·6827	·8748	·9652	·9936	·9993	
·33	·9999	·33	·0272	·1478	·3854	·6585	·8602	·9596	·9922	·9991	
·34	·9998	·34	·0238	·1339	·3610	·6338	·8447	·9533	·9906	·9989	0·9999
0·35	0·9998	0·35	0·0207	0·1211	0·3373	0·6089	0·8283	0·9464	0·9888	0·9986	0·9999
·36	·9997	·36	·0180	·1092	·3144	·5837	·8110	·9388	·9867	·9983	·9999
·37	·9996	·37	·0156	·0983	·2924	·5584	·7928	·9304	·9843	·9979	·9999
·38	·9996	·38	·0135	·0882	·2713	·5331	·7738	·9213	·9816	·9974	·9998
·39	·9995	·39	·0117	·0790	·2511	·5078	·7540	·9114	·9785	·9969	·9998
0·40	0·9993	0·40	0·0101	0·0705	0·2318	0·4826	0·7334	0·9006	0·9750	0·9962	0·9997
·41	·9992	·41	·0087	·0628	·2134	·4576	·7122	·8891	·9710	·9954	·9997
·42	·9990	·42	·0074	·0558	·1961	·4330	·6903	·8767	·9666	·9945	·9996
·43	·9988	·43	·0064	·0495	·1796	·4087	·6678	·8634	·9617	·9935	·9995
·44	·9986	·44	·0054	·0437	·1641	·3848	·6449	·8492	·9563	·9923	·9994
0·45	0·9983	0·45	0·0046	0·0385	0·1495	0·3614	0·6214	0·8342	0·9502	0·9909	0·9992
·46	·9980	·46	·0039	·0338	·1358	·3386	·5976	·8183	·9436	·9893	·9991
·47	·9976	·47	·0033	·0296	·1231	·3164	·5735	·8015	·9363	·9875	·9989
·48	·9972	·48	·0028	·0259	·1111	·2948	·5491	·7839	·9283	·9855	·9986
·49	·9967	·49	·0023	·0225	·1001	·2740	·5246	·7654	·9196	·9831	·9984
0·50	0·9961	0·50	0·0020	0·0195	0·0898	0·2539	0·5000	0·7461	0·9102	0·9805	0·9980

See page 4 for explanation of the use of this table.

TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 10$	$r = 0$	1	2	3	4	5	6	7	8	9
$p = 0.01$	0·9044	0·9957	0·9999							
·02	·8171	·9838	·9991							
·03	·7374	·9655	·9972	0·9999						
·04	·6648	·9418	·9938	·9996						
0·05	0·5987	0·9139	0·9885	0·9990	0·9999					
·06	·5386	·8824	·9812	·9980	·9998					
·07	·4840	·8483	·9717	·9964	·9997					
·08	·4344	·8121	·9599	·9942	·9994					
·09	·3894	·7746	·9460	·9912	·9990	0·9999				
0·10	0·3487	0·7361	0·9298	0·9872	0·9984	0·9999				
·11	·3118	·6972	·9116	·9822	·9975	·9997				
·12	·2785	·6583	·8913	·9761	·9963	·9996				
·13	·2484	·6196	·8692	·9687	·9947	·9994	0·9999			
·14	·2213	·5816	·8455	·9600	·9927	·9990	·9999			
0·15	0·1969	0·5443	0·8202	0·9500	0·9901	0·9986	0·9999			
·16	·1749	·5080	·7936	·9386	·9870	·9980	·9998			
·17	·1552	·4730	·7659	·9259	·9832	·9973	·9997			
·18	·1374	·4392	·7372	·9117	·9787	·9963	·9996			
·19	·1216	·4068	·7078	·8961	·9734	·9951	·9994	0·9999		
0·20	0·1074	0·3758	0·6778	0·8791	0·9672	0·9936	0·9991	0·9999		
·21	·0947	·3464	·6474	·8609	·9601	·9918	·9988	·9999		
·22	·0834	·3185	·6169	·8413	·9521	·9896	·9984	·9998		
·23	·0733	·2921	·5863	·8206	·9431	·9870	·9979	·9998		
·24	·0643	2673	·5558	·7988	·9330	·9839	·9973	·9997		
0·25	0·0563	0·2440	0·5256	0·7759	0·9219	0·9803	0·9965	0·9996		
·26	·0492	·2222	·4958	·7521	·9096	·9761	·9955	·9994		
·27	·0430	·2019	·4665	·7274	·8963	·9713	·9944	·9993	0·9999	
·28	·0374	·1830	·4378	·7021	·8819	·9658	·9930	·9990	·9999	
·29	·0326	·1655	·4099	·6761	·8663	·9596	·9913	·9988	·9999	
0·30	0·0282	0·1493	0·3828	0·6496	0·8497	0·9527	0·9894	0·9984	0·9999	
·31	·0245	·1344	·3566	·6228	·8321	·9449	·9871	·9980	·9998	
·32	·0211	·1206	·3313	·5956	·8133	·9363	·9845	·9975	9997	
·33	·0182	·1080	·3070	·5684	·7936	·9268	·9815	·9968	·9997	
·34	·0157	·0965	·2838	·5411	·7730	·9164	·9780	·9961	·9996	
0·35	0·0135	0·0860	0·2616	0·5138	0·7515	0·9051	0·9740	0·9952	0·9995	
·36	·0115	·0764	·2405	·4868	·7292	·8928	·9695	·9941	·9993	
·37	·0098	·0677	·2206	·4600	·7061	·8795	·9644	·9929	·9991	
·38	·0084	·0598	·2017	·4336	·6823	·8652	·9587	·9914	·9989	0·9999
·39	·0071	·0527	·1840	·4077	·6580	·8500	·9523	·9897	·9986	·9999
0·40	0·0060	0·0464	0·1673	0·3823	0·6331	0·8338	0·9452	0·9877	0·9983	0·9999
·41	·0051	·0406	·1517	·3575	·6078	·8166	·9374	·9854	·9979	·9999
·42	·0043	·0355	·1372	·3335	·5822	·7984	·9288	·9828	·9975	·9998
·43	·0036	·0309	·1236	·3102	·5564	·7793	·9194	·9798	·9969	·9998
·44	·0030	·0269	·1111	·2877	·5304	·7593	·9092	·9764	·9963	·9997
0·45	0·0025	0·0233	0·0996	0·2660	0·5044	0·7384	0·8980	·9726	0·9955	0·9997
·46	·0021	·0201	·0889	·2453	·4784	·7168	·8859	·9683	·9946	·9996
·47	·0017	·0173	·0791	·2255	·4526	·6943	·8729	·9634	·9935	·9995
·48	·0014	·0148	·0702	·2067	·4270	·6712	·8590	·9580	·9923	·9994
·49	·0012	·0126	·0621	·1888	·4018	·6474	·8440	·9520	·9909	·9992
0·50	0·0010	0·0107	0·0547	0·1719	0·3770	0·6230	0·8281	0·9453	0·9893	0·9990

See page 4 for explanation of the use of this table.

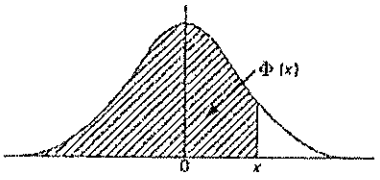
TABLE 1. THE BINOMIAL DISTRIBUTION FUNCTION

$n = 11$	$r = 0$	1	2	3	4	5	6	7	8	9	10
$p = 0.01$	0.8953	0.9948	0.9998								
.02	.8007	.9805	.9988								
.03	.7153	.9587	.9963	0.9998							
.04	.6382	.9308	.9917	.9993							
0.05	0.5688	0.8981	0.9848	0.9984	0.9999						
.06	.5063	.8618	.9752	.9970	.9997						
.07	.4501	.8228	.9630	.9947	.9995						
.08	.3996	.7819	.9481	.9915	.9990	0.9999					
.09	.3544	.7399	.9305	.9871	.9983	.9998					
0.10	0.3138	0.6974	0.9104	0.9815	0.9972	0.9997					
.11	.2775	.6548	.8880	.9744	.9958	.9995					
.12	.2451	.6127	.8634	.9659	.9939	.9992	0.9999				
.13	.2161	.5714	.8368	.9558	.9913	.9988	.9999				
.14	.1903	.5311	.8085	.9440	.9881	.9982	.9998				
0.15	0.1673	0.4922	0.7788	0.9306	0.9841	0.9973	0.9997				
.16	.1469	.4547	.7479	.9154	.9793	.9963	.9995				
.17	.1288	.4189	.7161	.8987	.9734	.9949	.9993	0.9999			
.18	.1127	.3849	.6836	.8803	.9666	.9932	.9990	.9999			
.19	.0985	.3526	.6506	.8603	.9587	.9910	.9986	.9998			
0.20	0.0859	0.3221	0.6174	0.8389	0.9496	0.9883	0.9980	0.9998			
.21	.0748	.2935	.5842	.8160	.9393	.9852	.9973	.9997			
.22	.0650	.2667	.5512	.7919	.9277	.9814	.9965	.9995			
.23	.0564	.2418	.5186	.7667	.9149	.9769	.9954	.9993	0.9999		
.24	.0489	.2186	.4866	.7404	.9008	.9717	.9941	.9991	.9999		
0.25	0.0422	0.1971	0.4552	0.7133	0.8854	0.9657	0.9924	0.9988	0.9999		
.26	.0364	.1773	.4247	.6854	.8687	.9588	.9905	.9984	.9998		
.27	.0314	.1590	.3951	.6570	.8507	.9510	.9881	.9979	.9998		
.28	.0270	.1423	.3665	.6281	.8315	.9423	.9854	.9973	.9997		
.29	.0231	.1270	.3390	.5989	.8112	.9326	.9821	.9966	.9996		
0.30	0.0198	0.1130	0.3127	0.5696	0.7897	0.9218	0.9784	0.9957	0.9994		
.31	.0169	.1003	.2877	.5402	.7672	.9099	.9740	.9946	.9992	0.9999	
.32	.0144	.0888	.2639	.5110	.7437	.8969	.9691	.9933	.9990	.9999	
.33	.0122	.0784	.2413	.4821	.7193	.8829	.9634	.9918	.9987	.9999	
.34	.0104	.0690	.2201	.4536	.6941	.8676	.9570	.9899	.9984	.9998	
0.35	0.0088	0.0606	0.2001	0.4256	0.6683	0.8513	0.9499	0.9878	0.9980	0.9998	
.36	.0074	.0530	.1814	.3981	.6419	.8339	.9419	.9852	.9974	.9997	
.37	.0062	.0463	.1640	.3714	.6150	.8153	.9330	.9823	.9968	.9996	
.38	.0052	.0403	.1478	.3455	.5878	.7957	.9232	.9790	.9961	.9995	
.39	.0044	.0350	.1328	.3204	.5603	.7751	.9124	.9751	.9952	.9994	
0.40	0.0036	0.0302	0.1189	0.2963	0.5328	0.7535	0.9006	0.9707	0.9941	0.9993	
.41	.0030	.0261	.1062	.2731	.5052	.7310	.8879	.9657	.9928	.9991	0.9999
.42	.0025	.0224	.0945	.2510	.4777	.7076	.8740	.9601	.9913	.9988	.9999
.43	.0021	.0192	.0838	.2300	.4505	.6834	.8592	.9539	.9896	.9986	.9999
.44	.0017	.0164	.0740	.2100	.4236	.6586	.8432	.9468	.9875	.9982	.9999
0.45	0.0014	0.0139	0.0652	0.1911	0.3971	0.6331	0.8262	0.9390	0.9852	0.9978	0.9998
.46	.0011	.0118	.0572	.1734	.3712	.6071	.8081	.9304	.9825	.9973	.9998
.47	.0009	.0100	.0501	.1567	.3459	.5807	.7890	.9209	.9794	.9967	.9998
.48	.0008	.0084	.0436	.1412	.3213	.5540	.7688	.9105	.9759	.9960	.9997
.49	.0006	.0070	.0378	.1267	.2974	.5271	.7477	.8991	.9718	.9951	.9996
0.50	0.0005	0.0059	0.0327	0.1133	0.2744	0.5000	0.7256	0.8867	0.9673	0.9941	0.9995

See page 4 for explanation of the use of this table.

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995		0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·759	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·791	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·826	0·99994	4·417	0·99999
			0·9999	3·867	0·99995		1·00000

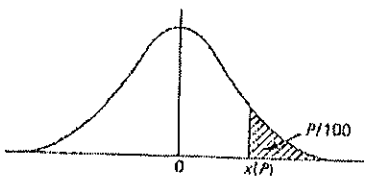
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteq \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172